UNIVERSITY OF TIKRIT
ENGINEERING COLLEGE
Mechanical & Chemical Engineering Department

Engineering Mechanics
Statics Lectures

Chapter four

Lecturer
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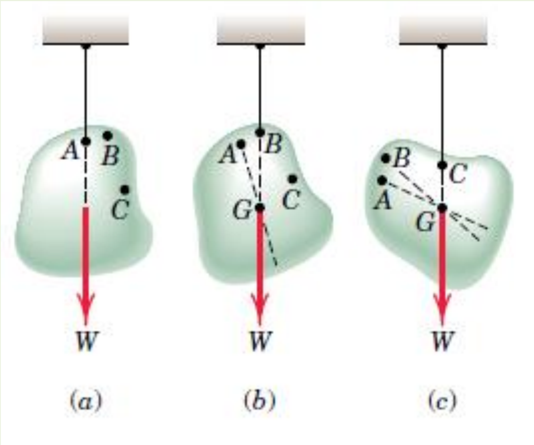
٢٠٢١- ٢٠٢٢

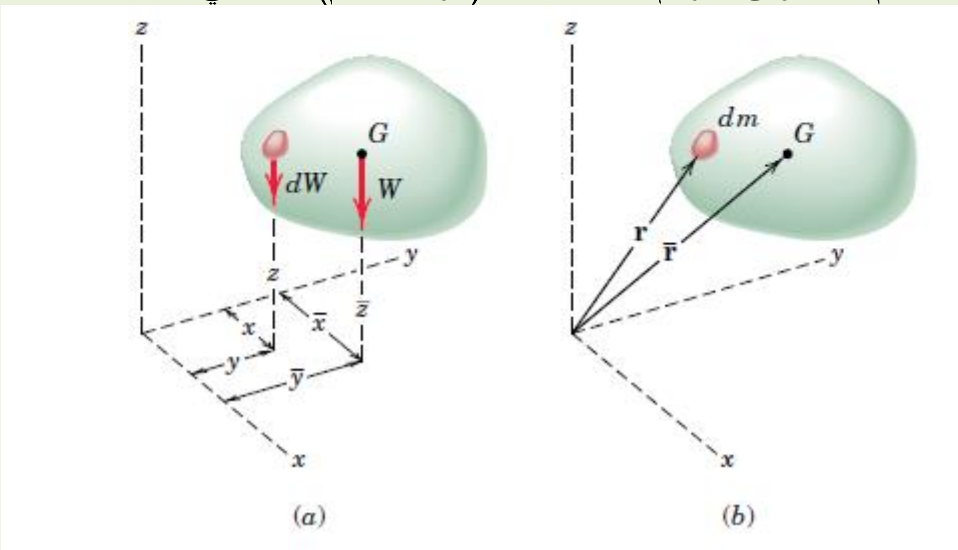
Chapter four

٤-١ CENTER OF MASS AND CENTROIDS

لايجاد مركز الثقل للاجسام مختبريا يتم اتباع الطريقة التالية:



اما في حالة ايجاد مركز الثقل رياضيا فيتم عن طريق العزوم عند اخذ القوة (وزن الجسيم) وكما يلي



To determine mathematically the location of the center of gravity of any body, Fig. ٥/٤*a*, we apply the *principle of moments* to the parallel system of gravitational forces. The moment of the resultant gravitational force *W* about any axis equals the sum of the moments about the same axis of the gravitational forces *dW* acting on all particles treated as infinitesimal elements of the body. The resultant of the gravitational forces acting on all elements is the weight of the body and is given by the sum *W* = *dW*. If we apply the moment principle about the *y*-axis, for example, the moment about this axis of the elemental weight is *x dW*, and the sum of these moments for all elements of the body is
= *x dW*. This sum of moments must equal the moment of the sum $W\bar{x}$. Thus, $\bar{x}W = \int x\,dW.$.
With similar expressions for the other two components, we may express the coordinates of the center of gravity *G* as

$$\bar{x} = \frac{\int x\, dW}{W} \qquad \bar{y} = \frac{\int y\, dW}{W} \qquad \bar{z} = \frac{\int z\, dW}{W}$$

Eq .....(٤-١)

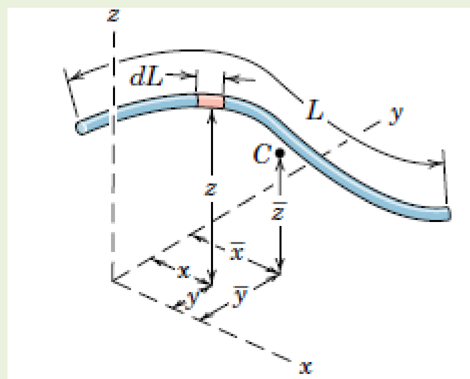
With the substitution of $W = mg$ and $dW = g\, dm$, the expressions for the coordinates of the center of gravity become

$$\bar{x} = \frac{\int x\, dm}{m} \qquad \bar{y} = \frac{\int y\, dm}{m} \qquad \bar{z} = \frac{\int z\, dm}{m}$$

.....(٤-٢)

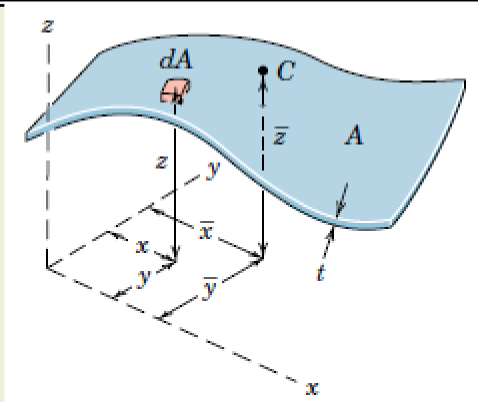
٤-٢ CENTROIDS OF LINES, AREAS, AND VOLUMES

(1) Lines. For a slender rod or wire of length *L*, cross-sectional area *A*, and density _, Fig. dowm, the body approximates a line segment, and (*dm* =*A dL)*. If _ and *A* are constant over the length of the rod, the coordinates of the center of mass also become the coordinates of the centroid *C* of the line segment, which, from Eqs. (٤-١) may be written



$$\bar{x} = \frac{\int x\, dL}{L} \qquad \bar{y} = \frac{\int y\, dL}{L} \qquad \bar{z} = \frac{\int z\, dL}{L}$$

(2) Areas. When a body of density _ has a small but constant thickness *t*, we can model it as a surface area *A*, Fig. ٥/٧. The mass of an element becomes *dm* =*t dA*. Again, if *p* and *t* are constant over the entire area, the coordinates of the center of mass of the body also become the coordinates of the centroid *C* of the surface area, and from Eqs. (٤-٢) the coordinates may be written as



$$\bar{x} = \frac{\int x\,dA}{A} \qquad \bar{y} = \frac{\int y\,dA}{A} \qquad \bar{z} = \frac{\int z\,dA}{A}$$

By the same ways for volume……

٤-٣ COMPOSITE BODIES AND FIGURES:

When a body or figure can be conveniently divided into several parts whose mass centers are easily de-termined, we use the principle of moments and treat each part as a finite element of the whole. Such a body is illustrated schematically in Fig. dwon. Its parts have masses *m*١, *m*٢, *m*٣ with the respective mass-center coordinates in the *x*-direction.

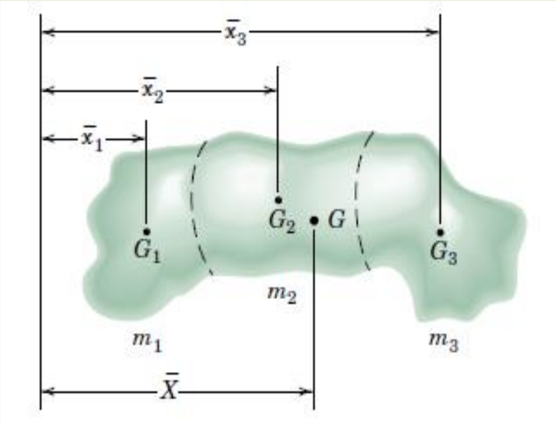
The moment principle gives

$$(m_1 + m_2 + m_3)\bar{X} = m_1\bar{x}_1 + m_2\bar{x}_2 + m_3\bar{x}_3$$

where is X the *x*-coordinate of the center of mass of the whole. Similar relations hold for the other two coordinate directions.

We generalize, then, for a body of any number of parts and express the sums in condensed form to obtain the mass-center coordinates

$$\bar{X} = \frac{\Sigma m\bar{x}}{\Sigma m} \qquad \bar{Y} = \frac{\Sigma m\bar{y}}{\Sigma m} \qquad \bar{Z} = \frac{\Sigma m\bar{z}}{\Sigma m}$$



This case for mass and for areas , lines and volumes its in same procedures.

ملاحظة:

في حل المسائل الخاصة في تحديد مراكز الثقل او المراكز الهندسية يجب اتباع الخطوات التالية:

اولاً: يتم تقسيم الشكل او الجسم الى اجزاء مبسطة ومعروفة (منتظمة).

ثانياً: يتم تحديد مركز كل جزء (احداثيات كل جزء نسبة الى نقطة الاصل).

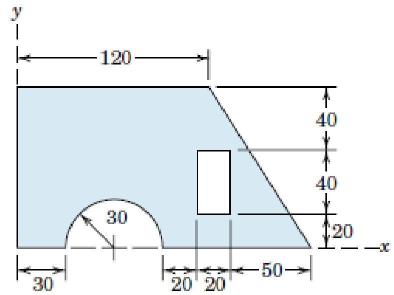
ثالثاً: كتابة الجدول الخاص باحتساب مراكز الثقل او المراكز الهندسية وكما مدون ادناه.

الشكل	A,L,m,V,W	X	Y	Z	AX	AY	AZ
المجموع	∑A or ∑m	----	---	---	∑AX	∑Ay	∑AZ

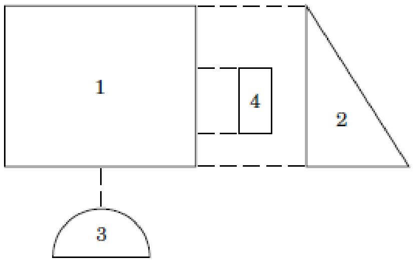
ثم ايجاد المراكز(اي الاحداثيات لمركز الجسم او الشكل المركب) حسب المعادلات

(٤-١),(٤-٢)......

Locate the centroid of the shaded area.



Dimensions in millimeters



Solution. The composite area is divided into the four elementary shapes shown in the lower figure. The centroid locations of all these shapes may be obtained from Table D/3. Note that the areas of the "holes" (parts 3 and 4) are taken as negative in the following table:

PART	A mm^2	$\bar{x}$ mm	$\bar{y}$ mm	$\bar{x}A$ mm^3	$\bar{y}A$ mm^3
1	12 000	60	50	720 000	600 000
2	3000	140	100/3	420 000	100 000
3	−1414	60	12.73	−84 800	−18 000
4	−800	120	40	−96 000	−32 000
TOTALS	12 790			959 000	650 000

The area counterparts to Eqs. 5/7 are now applied and yield

$$\left[\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A}\right] \qquad \bar{X} = \frac{959\,000}{12\,790} = 75.0 \text{ mm} \qquad \textit{Ans.}$$

$$\left[\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}\right] \qquad \bar{Y} = \frac{650\,000}{12\,790} = 50.8 \text{ mm} \qquad \textit{Ans.}$$

Locate the centroid of the wire shown in Fig. 9–16*a*.

SOLUTION

Composite Parts. The wire is divided into three segments as shown in Fig. 9–16*b*.

Moment Arms. The location of the centroid for each segment is determined and indicated in the figure. In particular, the centroid of segment ① is determined either by integration or by using the table on the inside back cover.

Summations. For convenience, the calculations can be tabulated as follows:

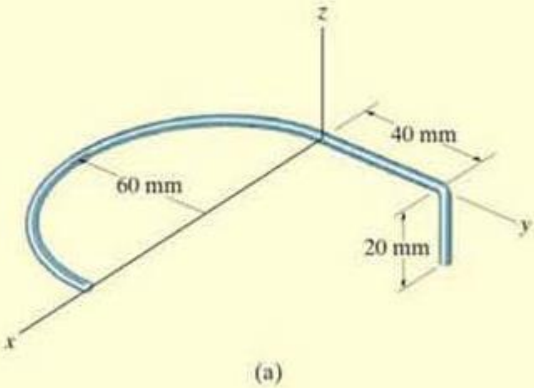
Segment	L (mm)	$\tilde{x}$ (mm)	$\tilde{y}$ (mm)	$\tilde{z}$ (mm)	$\tilde{x}L$ (mm^2)	$\tilde{y}L$ (mm^2)	$\tilde{z}L$ (mm^2)
1	$\pi(60) = 188.5$	60	−38.2	0	11 310	−7200	0
2	40	0	20	0	0	800	0
3	20	0	40	−10	0	800	−200
	$\Sigma L = 248.5$				$\Sigma\tilde{x}L = 11\,310$	$\Sigma\tilde{y}L = -5600$	$\Sigma\tilde{z}L = -200$

Thus,

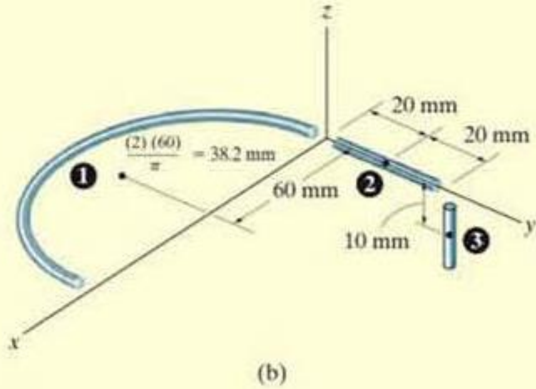
$$\bar{x} = \frac{\Sigma\tilde{x}L}{\Sigma L} = \frac{11\,310}{248.5} = 45.5 \text{ mm} \qquad \textit{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}L}{\Sigma L} = \frac{-5600}{248.5} = -22.5 \text{ mm} \qquad \textit{Ans.}$$

$$\bar{z} = \frac{\Sigma\tilde{z}L}{\Sigma L} = \frac{-200}{248.5} = -0.805 \text{ mm} \qquad \textit{Ans.}$$



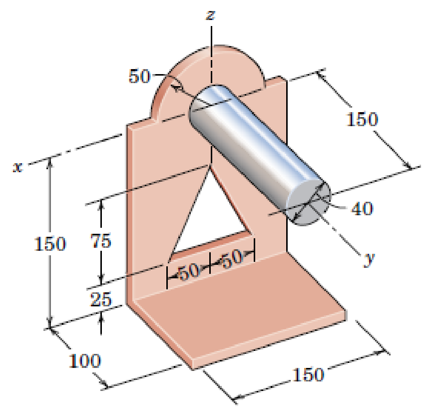
(a)



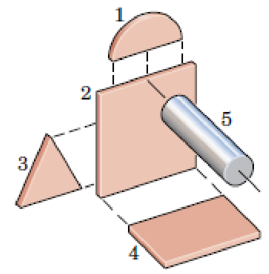
(b)

Sample Problem 5/8

Locate the center of mass of the bracket-and-shaft combination. The vertical face is made from sheet metal which has a mass of 25 kg/m^2. The material of the horizontal base has a mass of 40 kg/m^2, and the steel shaft has a density of 7.83 Mg/m^3.



Dimensions in millimeters



Solution. The composite body may be considered to be composed of the five elements shown in the lower portion of the illustration. The triangular part will be taken as a negative mass. For the reference axes indicated it is clear by symmetry that the x-coordinate of the center of mass is zero.

The mass m of each part is easily calculated and should need no further explanation. For Part 1 we have from Sample Problem 5/3

$$\bar{z} = \frac{4r}{3\pi} = \frac{4(50)}{3\pi} = 21.2 \text{ mm}$$

For Part 3 we see from Sample Problem 5/2 that the centroid of the triangular mass is one-third of its altitude above its base. Measurement from the coordinate axes becomes

$$\bar{z} = -[150 - 25 - \tfrac{1}{3}(75)] = -100 \text{ mm}$$

The y- and z-coordinates to the mass centers of the remaining parts should be evident by inspection. The terms involved in applying Eqs. 5/7 are best handled in the form of a table as follows:

PART	m kg	$\bar{y}$ mm	$\bar{z}$ mm	$m\bar{y}$ kg·m	$m\bar{z}$ kg·mm
1	0.098	0	21.2	0	2.08
2	0.562	0	−75.0	0	−42.19
3	−0.094	0	−100.0	0	9.38
4	0.600	50.0	−150.0	30.0	−90.00
5	1.476	75.0	0	110.7	0
TOTALS	2.642			140.7	−120.73

Equations 5/7 are now applied and the results are

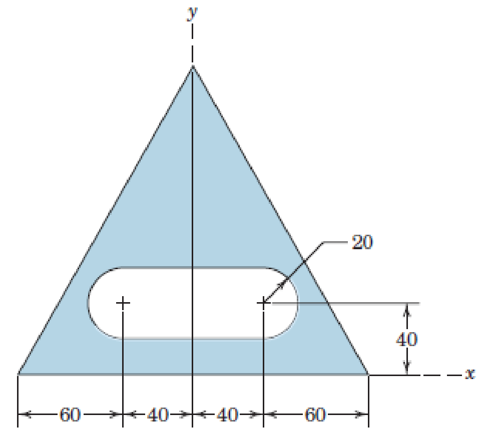
$$\left[\bar{Y} = \frac{\Sigma m\bar{y}}{\Sigma m}\right] \qquad \bar{Y} = \frac{140.7}{2.642} = 53.3 \text{ mm} \qquad \textit{Ans.}$$

$$\left[\bar{Z} = \frac{\Sigma m\bar{z}}{\Sigma m}\right] \qquad \bar{Z} = \frac{-120.73}{2.642} = -45.7 \text{ mm} \qquad \textit{Ans.}$$

Probs:

(١)

Determine the y-coordinate of the centroid of the shaded area. The triangle is equilateral.

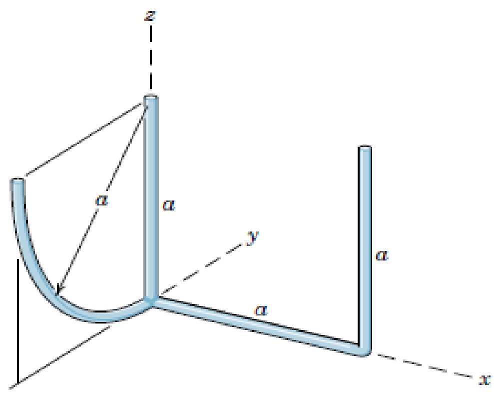


Dimensions in millimeters

(٢)

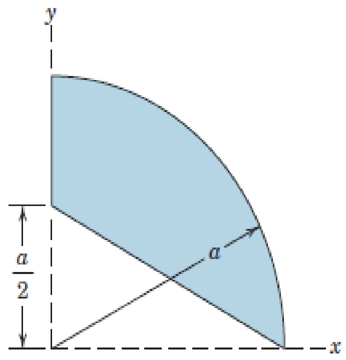
Determine the coordinates of the mass center of the welded assembly of uniform slender rods made from the same bar stock.

Ans. $\overline{X} = \frac{3a}{6+\pi}$, $\overline{Y} = -\frac{2a}{6+\pi}$, $\overline{Z} = \frac{\pi a}{6+\pi}$



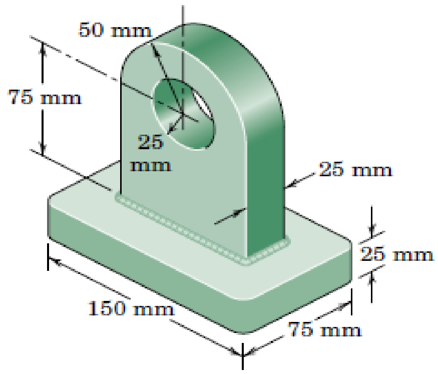
(٣)

Determine the x- and y-coordinates of the centroid of the area of Prob. 5/26 by the method of this article.



(٤)

Determine the distance $\overline{H}$ from the bottom of the base to the mass center of the bracket casting.



(٥)

Locate the centroid $(\bar{x}, \bar{y}, \bar{z})$ of the wire which is bent in the shape shown.